University of Saskatchewan

Department of Computational Science
CMPT 496 Final
April 14, 1990

For your own convenience, and to make marking easier, please begin each question at the top of a new page.

Marks (Approx.)

6 1. A queue has a service rate of 25 customers per hour. The queue starts empty at 8:00, and the number of arrivals from 8:00 to 9:00 are 40. From 9:00 to 10.00, the number of arrivals to the queue are 10. Find the average queue length and the average waiting time during the time-interval from 8:00 to 10:00.

3 2. Why is it necessary to standardize the probabilities such that the sum of all probabilities in each center of an open network equals 1. Why is it unnecessary to do this standardization in a closed network. If possible, give a mathematical proof. Partial marks will be given for intuitive reasoning.

3. An assembly line consists of an M/M/3 queue, followed by an M/D/1 queue. The arrival rate to the M/M/3 queue is 8 per hour, and the average service time is 5 minutes. 20% of the output of the M/M/3 queue is defective, and it is returned to the M/M/3 queue, where it is treated like all other arrivals. As the notation M/D/1 queue implies, the service time of the second station is constant, and it is 4 minutes. (Note that since the output of the M/M/3 queue is Poisson, so is the input to the M/D/1 queue)

5 a) Find the probability that all servers of the M/M/3 queue are busy.

5 a) Find the probability that over 2 products are waiting in front of the first station, excluding the ones that are presently worked at.

4 b) Find the average number of items in the first queue.

11 c) Find the probability that exactly 1 item is waiting in front of the second station, not counting the item being worked at.

6 d) Find the total number of elements in the system. Also, find the average time a part spends in the system.

4. In a plant, there are three machines. The machines have to be refilled with raw material, and the time between two refills is exponentially distributed with an average of 3 hours. The refill is done by two employees, who form a two server queue, and the average time is 1 hour per refill. After each refill, the machine is started up again. There is, however, a probability of 0.2 that the machine does not start properly, and that it has to be repaired. There is one repairman, and the average repair time is 4 hours. After repair, the machine is running again. All times in question follow an exponential distribution.

16 a) Find the distribution of the number of machines waiting to be filled.

5 b) Find the percentage of time the two employees which are in charge of filling are busy.

6 c) Find the average time a machine has to wait for filling.

6 d) Find the probability that 2 machines are being refilled, one machine is waiting for feeding, no machine is in repair, and that the remaining machine is running.

Note: in case you were unable to solve question a, make up your own figures, and continue with question b, c and d.

12 5. A cyclic queue consists of four one-server queues, all of which have the same exponential service time. There are five elements in the cyclic queue.
Use generating functions to find the convolution of the number of elements in the first 3 queues. Use this result to find the distribution of the number of elements in the forth queue.
Note: In a cyclic queue, all centers are visited in the order center 1, center 2, ..., center n, and after that, the item returns again to center 1.

12 6) A semi-Markov chain has three states. The expected time in state 1 is 3 hours, the expected time in state 2 5 hours, and the expected time in state 3 2 hours. The transition probabilities at the time of a state change are as follows

	1	2	3
1	0.4	0.2	0.6
2	0.5	0.5	0.0
3	0.1	0.7	0.2

a) Find the expected number of visits to the different states between two visits of state 1.

b) Find the equilibrium probability to be in state 3 at a time chosen at random.

c) Find the expected time spent in state 3 between ~~to~~ two visits to state 1.

100

********* The End ********